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ABSTRACT

The dual purpose of this study was (1) to design instruction in problem representation and solution of algebraic word problems in order to improve the algebraic problem solving skills of learning disabled (LD) adolescents; and (2) to devise and evaluate the effectiveness of instructional procedures for teaching LD adolescents to represent and solve algebra word problems. Twenty learning disabled students (Grades 8-10) were assigned to instructed (experimental) or control conditions. The experimental group received guided instruction in problem representation and problem solution for 600 algebra problems representing each of three mathematical structures: relational, proportion, and two-variable two-equation problems. Elements of the instructed phase included six pretests, intervention on alternate days for 12 weeks (using techniques which included scripts, verbal prompts, self-questioning, and reinforcement), six posttests, tests of near-transfer and far-transfer, and maintenance evaluation. The control groups received pretests, posttests, and their regular program of mathematics instruction. Examination of the single-subject data indicated that instructed students learned the three problem types, maintained what they had learned after a 6-week interval, and transferred their learning to problems deliberately constructed to be different. For two of three measures used, instructed students had significantly higher outcome scores than controls. (JW)

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TEACHING REPRESENTATION AND SOLUTION OF ALGEBRAIC WORD PROBLEMS TO LEARNING DISABLED ADOLESCENTS

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Introduction

The topic addressed in the research reported in this paper is problem solving, particularly word problems that can be found in mathematics curricula.

Usually, a mathematical word problem identifies quantities, describes relationships between them, situates this information in a familiar context, and requires that the student attain a goal. Algebra word problems, in particular, require the use of symbols to represent unknown quantities, and the solution of equations to derive the value of these symbols before the goal can be attained. Recently, educators have expressed concern about adolescents' inability to understand and solve algebraic word problems (Carpenter, Corbitt, Kepner, Lindquist & Reyes, 1980). The learning disabled are one group of adolescents who seldom succeed in mastering algebra word problems (Fleischner & Garnett, 1983). These students have ability in the average range and significant discrepancy between their ability and their low achievement in particular curricular areas such as reading or mathematics. Although many students fail to master algebra, solving word problems is frequently necessary for completion of mathematics requirements in secondary and post-secondary programs.

Recent theoretical formulations about the nature of problem solving yield prescriptions that may be instrumental in enabling educators to meet their aims in problem solving instruction. Problem solving in knowledge-rich domains such as algebra consists of two phases: problem representation and problem solution (Mayer, 1985). Problem representation refers to "getting the whole picture," understanding the relationships given in the problem, and generating the equations. Problem solution refers to solving the equations, and reaching the goal, or answering the question posed in the problem. Recent reviews demonstrate that experts are better than novices at representation, while there are few differences in ability to solve once representations have been generated (Glaser, 1984; Hutchinson, 1985). Studies of the mathematical problem solving of learning disabled adolescents have demonstrated that they are unable to understand the relationships expressed in word problems and fail to construct problem representations (e.g., Lee & Hudson, 1981). In spite of those findings which suggest that failure to represent is primarily responsible for difficulty with word problems, most instruction has focused on teaching problem solution (Simon, 1980).

The Instructional Study

The instructional study described here can be thought of as having two challenges: design of <u>what</u> was taught, and <u>how</u> it was taught. The theoretical significance lies in <u>what</u> was taught: the knowledge that was presented, coupled with the particular way in which it was structured. The structure was derived from current theory in cognitive instructional psychology.

The purposes of the study match the two challenges just mentioned. The first purpose, concerning the knowledge, and the second, concerning the effectiveness of the instruction, are described separately below, each with its rationale.

Purpose 1: Knowledge and Instructional Design

Importantly, the first purpose of this study is understood in terms of specific instructional theory and derived techniques, as explained below. The first purpose was to design instruction in problem representation as well as problem solution of algebraic word problems to improve the algebraic problem solving of learning disabled adolescents. In order to teach complex cognitive tasks, it is necessary to break them down into the component cognitive processes and knowledge structures essential for competent understanding and performance. Because experts think and perform fluently and quickly, their performance is insufficient as a model for instruction. Theoretical cognitive task analyses carried out on mechanics problems in physics (Heller & Reif, 1984) were employed as a model to determine the declarative and procedural knowledge that would be instructed for each of three types of algebra word problems commonly found in junior high school curricula.

It has been advocated that what must be taught to enable students to construct problem representations are schemata for the most important types of algebra word problems. There are two major systems for classifying types of algebra word problems: by the form of the underlying equations, and by the general form of the story line. The form of the underlying equation is based on the mathematical relations present in the problem and refers to the structure of the solution equation. For example, the standard solution for a problem may require the solving of two equations in two unknowns (simultaneous linear equations). The form of the story line refers to contextual or surface details such as mention of money, age, or river current.

For many problems in the algebra curriculum, there is no necessary connection between the problem structure and the surface structure. Money can be associated with simultaneous linear equations or proportion problems. However, many of the investigations which provide the strongest arguments in favour of instruction to develop problem schemata confound the mathematical structure and the story line (e.g., Hinsley, Hayes, & Simon, 1977). Mayer (1981) argued that instruction in algebra problem solving should be constructed around "templates." A template refers to a specific propositional structure in combination with a particular story line. He reasoned that because "...in general certain major categories of problems (based on story line) involved characteristic underlying equations", it is important to combine the two. This is the case only in formula-based problems, however. Students who receive instruction for templates must then be taught to generalize from the template to other problems with the same mathematical-propositional structure and new story lines (known as problem isomorphs). Isomorphs are problems in which the solution paths map directly on to one another in one-to-one fashion (Simon & Hayes, 1976). It would appear more effective and parsimonious to teach isomorphs rather than templates.

Three problem types or isomorphs were chosen for instruction in the study. Each is characterized by the mathematical relationships that have to be understood in order to construct a thorough and accurate representation, and by the form of the equation, derived from the representation, that has to be solved. The three problem types are relational problems, proportion problems, and problems in two variables and two equations. An example of a relational

problem is: "Sam has $18 more than Tom. Together they have $82. Find how much money each boy has." A proportion problem is: "Brian saved $50 in 18 weeks. At that rate, how long will it take him to save $350?" An example of a problem in two variables and two equations is: "Andrew has 18 coins, some quarters and some dimes. The total value of the coins is $3.45. Find the number of each kind of coin." Five story lines or surface features are used with each problem type. These story lines concern money, age, distance, work, and number.

In summary, for three problem isomorphs (Simon & Hayes, 1976), declarative and procedural knowledge were structured and sequenced for instruction in each of representation and solution, separately (Mayer, 1985), based on cognitive task analyses (Reif & Heller, 1984).

Purpose 2: Instructional Effectiveness

The second purpose was to devise and evaluate the effectiveness of instructional procedures for teaching learning disabled adolescents to represent and solve algebra word problems. Recent intervention studies (e.g., Nuzum, 1983) have employed guided instruction based on principles of cognitive behavior modification (Meichenbaum, 1977). They have been effective in teaching learning disabled adolescents mathematical word problems requiring one operation or two operations, and involving representations that require primarily a choice of operations. The procedures employed in these studies include modeling procedural knowledge by thinking aloud, guided practice in thinking aloud, and independent practice. Students use a self-questioning strategy to guide their procedural thinking.

The cognitive task analyses (Reif & Heller, 1984) suggested that use of declarative knowledge (terms and concepts) would have to be modeled as well as use of procedural knowledge. The complexity of the processing and structures required for problem representation suggested that students be cued. Detailed feedback provided during the course of thinking aloud, and at the close of each independent practice, is common to effective problem solving instruction for the learning disabled (for a review, see Hutchinson, 1986).

In summary, the components common to effective instructional studies in problem solving with learning disabled students were modified to take account of the complex declarative knowledge that cognitive task analyses (Reif & Heller, 1984) indicated would be necessary for competent representation and solution of algebraic word problems.

Method

Design of the Study

There were two overlapping designs employed in this study: a traditional experimental/control two-group design and a single-subject design. In the two-group design (see Figure 1), the experimental (instructed) group's performance was compared with a control group on several dependent measures, including instructed problem types and two general problem-solving tests. (Data on understanding shown in think-aloud protocols, a metacognitive interview, and

a problem classification task are presented in Hutchinson (1986)). In the single-subject design, the course of intervention was recorded for individual students. Maintenance was measured six weeks after the end of instruction. Tests of near- and far-transfer were also administered. Near-transfer problems used the same mathematical structure with new cover stories, while far-transfer problems used the same cover stories and a more complex variation of the mathematical structure.

Instructed problems were scored for thorough and accurate representation and solution, as well as for correct numerical answers using criteria based on prescriptive task analysis.

Subjects

Twenty students from grades eight, nine, and ten took part in this study. The twenty were nominated by their learning assistance teachers as adequate in basic operations of mathematics but poor at solving abstract mathematical word problems. These students had been identified as learning disabled during their elementary school years in accord with school district guidelines and the guidelines of the province of British Columbia in Canada.

To confirm the teachers' nominations, and to specify the task domain to which criterion-referenced measures of learning outcomes would be applied (Gagné & Beard, 1978), we administered criteria tests of mathematical operations and word-problem solving. To meet the criterion of adequate performance in mathematical operations, the students were required to score at the standard mastery criterion of 80% correct answers on a ten-minute, 35-item test of addition, subtraction, multiplication, and division with whole numbers. The second criterion required familiarity or 60% correct answers in a set of five one-step word problems requiring the operation of multiplication. Next, students' difficulties with algebraic word problems were established by determining that they met one criterion: no more than 40% correct numerical answers on an experimental-task pretest of 15 problems.

All students from two schools who met these criteria were invited to participate, and were assigned randomly to two conditions: 12 to the instructed condition and 8 to the control condition. The 20 student participants are described in Table 1 according to group, age, sex, school, or grade, and performance criterion measures. The two groups were quite closely matched. Hotelling's $T2$ test was used to compare means of the instructed and comparison groups on the three criterion variables for inclusion in the study, yielding no significant difference $F(3,26)=.03$, $p > .05$.

Materials

A total of 600 problems was used in this study, 200 containing each of the three mathematical structures: relational, proportion, and two-variable two-equation problems. The 200 problems of each mathematical structure were made up of 40 of each of five surface structures or story lines: money, age, distance, work, and number. About one-quarter of the 600 problems were drawn from mathematics series used by Grade 8 teachers (Ebos, Robinson & Tuck, 1984; Eicholz, O'Daffer, Brumfiel, Shanks & Fleenor, 1971; Sobel & Maletsky, 1974;

Stein, 1982). The remaining three-quarters were prepared for this study. These problems were modelled on those from the texts, but they systematically combined each of the five surface structures with each of the three mathematical structures.

The 600 problems were assigned randomly to task sheets for training materials, baseline tests, and dependent measures for pre- and post-tests. Twenty tasksheets containing five problems each were prepared for each type of problem, and each tasksheet had one problem with each surface structure. Tasksheet #1 for relational problems appears in Figure 2. The odd-numbered tasksheets were used for training/practice, and the even-numbered tasksheets were used for regular task assessment. All students received the same training materials in the same sequence, but progressed at their own rate. The baseline tests consisted of ten problems of one type (two of each surface structure); the measures for pre- and posttest consisted of 15 problems (one of each mathematical structure/surface structure combination). All materials were examined by experienced mathematics educators, and were found to be free of misconceptions and internal inconsistencies, and to contain the intended mathematical and surface structures.

Procedures

Control Group

The control group received six pretests and posttests, and were made familiar with the structured worksheet used by the instructed students throughout instruction (see Figure 3). They received this familiarization individually in three 30-minute sessions. In each session, they were given explanations of the components of problem representation and problem solution that appeared on the worksheet. They were asked to define the terms and find the relevant components in problems on the first tasksheet for one type of problem: relational in the first session, proportion in the second session, and two-variable two-equation problems in the third session. They did not receive instruction or feedback specific to the problem types. These sessions took place in the week prior to posttesting. During the three months of the intervention, they received their regular program of instruction in mathematics and problem solving with their learning assistance teacher. Total instructional time for the students in the two groups was the same.

Instructed Group

Instead of their regular learning assistance program in mathematics and problem solving, the students in the instructed group received guided instruction in problem representation and problem solution for three successive problem types. For the instructed group, there were a number of aspects to the study: (1) six pretests over three sessions; (2) intervention on alternate days for 12 weeks (see Figure 4 for phases in intervention); (3) six posttests over three sessions; (4) transfer, consisting of tests of near transfer and far transfer over two to four sessions; (5) maintenance, consisting of the administration six weeks later of the pretest/posttest measure on the instructed task.

All sessions took place in small teaching rooms used by the learning assistance teachers in each school. Sessions were scheduled for 40 minutes on alternate days from November 1 to March 15, and for two days during the last week of April.

Intervention

Intervention consisted of a number of phases (as illustrated in Figure 4). The first major division was into types of problems: in order, relational problems, proportion problems, and problems in two variables and two equations. Within each problem type, instruction in representation preceded instruction in solution, and instruction in these two phases was bracketed by the administration of an audio taped thinkaloud.

Scripts were used to introduce each phase and provide students with relevant declarative and procedural knowledge: orientation script, general script about word problems, and specific scripts for representation and solution for each problem type. These can be found in Hutchinson (1986).

The general daily procedures used throughout intervention were:

1. The student was reminded of the purpose of the current session, and shown the graph of the results to date.

2. The tasksheet was placed in front of the student, with the prompt card for self-questioning. (The prompt was gradually faded out.)

3. The student read and said from memory the self-questions. The student and the teacher each read the first problem silently.

4. The instructor read aloud and thought aloud for the first word problem, making frequent references to the prompt card for self-questioning. This procedure was repeated for the second problem.

5. The student read aloud and thought aloud for the third word problem. The instructor prompted, encouraged, and provided feedback specific to the student's verbalizations. Examples are: "Are you certain that you are letting X take the place of the simplest unknown?" "That is correct, it is a relational problem; now, can you tell me what structural features of the problem helped you to identify it?" This procedure was repeated for the fourth problem.

6. The student completed the fifth word problem without interjection from the instructor, aloud in the early stages of the study, and then silently. Feedback was provided.

7. The frequent prompts, encouragement and feedback were gradually faded out.

8. After completing a practice tasksheet in this manner, the student completed an assessment tasksheet independently, seeking help only if unable to proceed.

9. The student was shown a graph of progress and informed about the purpose of the next session.

The self-questions on the prompt-card for representation were:

1. Have I read and understood each sentence? Are there any words whose meaning I have to ask?

2. Have I got the whole picture, a representation, for this problem?

3. Have I written down my representation on the worksheet? (goal; unknown(s); known(s); type of problem; equation)

4. What problem features should I focus on in a new problem so I can know whether I can use the representation I have been taught?

The self-questions on the prompt-card for solution were:

1. Have I written an equation?

2. Have I expanded the terms?

3. Have I written out the steps of my solution on the worksheet? (collected like terms; isolated unknown(s); solved for unknown(s); checked my answer with the goal; highlighted my answer)

4. What problem features should I focus on in a new problem so I can know whether I can use the solution I have been taught

Thinkaloud Protocols

Students were instructed to think aloud, while representing and solving on the structured worksheet, prior to baseline for representation and following attainment of criterion for solution in each problem type. These verbalizations were audiotaped, transcribed, and rated for degree of understanding shown in representation and solution. Consistent with recommendations made by Ericsson and Simon (1984), the protocol analyses were based on cognitive task analyses of each of the three types of problems. Guidelines used to determine ratings of degree of understanding are presented in Hutchinson (1986).

Transfer Tasks

To minimize the confusion between transfer and maintenance (Brown & Campione, 1984), the transfer tasks were administered to instructed students for only the problem types they had mastered, immediately after the posttests. There were 5 problems for each problem type for each of near-transfer and far-transfer conditions. The criterion for each set of transfer problems was 80%, as in the intervention.

The transfer tasks were designed according to information provided by the cognitive task analyses and consisted of two kinds of systematic variation from the instructed tasks. Each kind of variation was consistent with one of the major

systems for classifying types of algebra word problems. In near-transfer tasks, the general form of the story line was varied, while the mathematical structure was held constant. The story lines changed from age, work, number, money, distance to volume, mass, area, election results, test results. In far-transfer tasks, the mathematical structure, that is, the form of the underlying equation, was an elaboration on the instructed structure. One more cognitive operation was introduced. Equations for the far-transfer relational problems took the form $(x + 2x + 2) = 14$, rather than either $x + (y + 2) = 12$ or $x + 2x = 12$. Far-transfer proportion tasks took the form $5/10 = x/15 = y/25$ instead of $5/10 = x/15$. Far-transfer equations for problems in two variables and two equations took the form $x + y + z = 10$, and $2x + 3y + 5z = 31$, but it would be given that $x + y = 9$, and the student could deduce that $z = 1$. The instructed problems had equations in the form $x + y = 15$, $2x + 3y = 40$. In these tasks, while the form of the mathematical structure was varied, the storyline was held constant.

Pretests and Posttests

These measures were administered to instructed and control group students. In all cases, students were provided with a calculator and given as much time as they required.

A 15-item test consisting of five problems of each type, one of each story line, assessed directly the thoroughness and accuracy with which students completed the instructed problem types. Each problem was completed on a structured worksheet, and responses were scored according to the three sets of criteria used during intervention.

A multiple-choice measure prepared by the Ministry of Education in British Columbia, Canada (Robitaille, Sherrill, Kelleher, Klassen & O'Shea, 1980) contains three subtests related to instruction in this study: Analyzing Word Problems (8 items), Use of Diagrams (7 items), and Solving Problems (10 items). Only multiple-choice responses were scored, but students were asked to show their work.

An open-ended measure developed by the Learning Assessment Branch (1985) British Columbia Ministry of Education, was used as a measure of general problem-solving ability, although none of the 13 problems selected was an isomorph of the instructed problems. Only numerical answers were scored, although students were asked to show their work.

Maintenance Test

The maintenance data were collected six weeks after the end of instruction by administering the same 15-item pre-post assessment measure (containing five problems of each type of instructional problem) under the same conditions used on the two previous administrations.

Results

All between-groups and within-subject hypotheses were substantiated (Hutchinson, 1986). Single-subject data showed the instructed students' problem solving ability increased dramatically for each type. These increases

were both maintained and transferred. The data from between-group comparisons demonstrated the superiority of the instructed group in instructed problems and problem solving in general.

Daily Problem-Solving Assessments: Instructed Problems

The twelve instructed students progressed from one problem type to the next, after reaching criterion. Because students reached criterion at different rates, there was variation in the number of problem types they learned. Six students mastered all three problem types, ten students mastered the first two problem types, and two students mastered only one problem type.

The results for individual students on representation, solution, and answers for the first problem type (relational) are presented in aggregate form in Figure 5. Each student received two administrations of baseline (each consisting of ten questions) prior to instruction in representation and prior to instruction in solution. These were scored for each of representation and solution. The numerical answer, which is often thought of as the last step in solution, was scored separately. A student who reached criterion (80% correct) on both baseline measures for a phase received no instruction in that phase and progressed to the next baseline. When a student reached criterion on baseline, no instruction for that problem type was given. Following instruction, all students achieved criterion on the first problem type (relational), so Figure 5 represents the aggregate performance of all students. The number of assessments within instructional phases varied because students continued in a phase until they reached the criterion of 80% correct on three successive assessments. Posttests were administered at the end of three months of instruction, followed by transfer. Maintenance was tested six weeks later. The criteria for maintenance and transfer tasks were also set at 80%.

For every instructed student, there was dramatic improvement from baseline to assessments during instruction, and these dramatic improvements were maintained six weeks after the end of instruction.

Maintenance

Six weeks after the end of instruction, students were given the 15-item test that had been administered as a pretest and posttest. They had never received feedback about their performance on this measure of instructed problem types. Each problem was scored for representation, solution, and correct answer. Figure 5 shows graphically that the group of instructed students, as a whole, maintained what they had learned about relational problems, scoring above the criterion of 80% on each of representation, solution, and correct numerical answers. Examination of these single subject data for each type of instructed problem shows that most individual students maintained what they had learned. Six weeks after the end of instruction, criterial performance was maintained by 10 of the 12 students who had reached criterion on relational problems. For proportion problems, criterion was maintained by all ten students who had reached criterion during intervention. Five of the six students who reached criterion for two-variable two-equation problems maintained criterial performance six weeks later. The students' abilities to do the instructed problems improved, and were maintained over time.

Transfer

Instructed students received transfer tasks at the end of instruction only for the problem types on which they had reached criterion during instruction. The control students received no transfer tasks. The transfer tasks were deliberately constructed to differ from the instructed problems on only one dimension, either the mathematical structure or the surface structure.

Near-Transfer

Near-transfer tasks consisted of problems differing on only surface structure. Every student who reached criterion on proportion and two-variable two-equation problems reached criterion for these problem types on the near-transfer tasks for both representation, solution and answers (see Table 2). Every student showed near transfer for representation of relational problems. One student did not reach criterion on solution of near-transfer relational problems, and three students did not record correct numerical answers. In summary, the instructed students did show near-transfer to problems with the same mathematical structure, but with different surface structure or story line than the instructed problems.

Far-Transfer

Far-transfer tasks consisted of problems differing on mathematical structure only. These tasks were composed of problems containing the same surface structure as the instructed problems (money, age, distance, work and number) but, in each case, the mathematical structure had been made slightly more complex through the addition of one more mathematical proposition. Fewer students reached criterion on far-transfer tasks than near-transfer. Of the twelve students who reached criterion on representation, solution, and answers for relational problems in instruction, only four reached criterion on these aspects of the far-transfer problems. For proportion problems, most students reached criterion on far-transfer: eight out of ten on representation, nine out of ten on solution, and nine out of ten on answers. For problems in two-variables and two-equations, most students reached criterion on far-transfer: four out of six on representation, four out of six on solution, and four out of six on answers. In summary, the instructed students showed far-transfer on proportion and two-variable two-equation problems, but little far-transfer on relational problems. Additionally, problems described as far-transfer were more difficult than problems described as near-transfer.

Thinkaloud Protocols

Thinkaloud protocols for representation showed shifts from mere reading comprehension of problem statements to expression and application of domain-specific algebraic knowledge. Similarly, for solution, students moved from use of arithmetic operations to use of sophisticated algebraic procedures. There was almost no overlap in distributions of scores for thinkalouds before and after instruction in either representation or solution for each problem type. Verbalizations became more abstract, more complete, and more accurate, showing increased understanding of representation and solution.

Problem-Solving Comparison of Instructed and Control Students

Examination of the single-subject data demonstrates that the instructed students learned the three problem types for which they received instruction, maintained what they had learned after a six-week interval, and transferred this newly-acquired problem-solving ability to problems deliberately constructed to be different. The question remains whether they would have learned as much about problem solving if they had continued in their learning assistance programs for mathematics and problem solving. To answer this question, three measures of problem solving were administered as pretests and posttests to the instructed and control students: 15 instructed problems completed on structured worksheets, a multiple-choice and an open-ended test of junior high mathematical problem solving.

The 15-item pre-posttest was a criterion-referenced measure on which scores were not expected to be normally distributed. Fisher's Exact Test, a non-parametric statistic, was used to answer whether the two groups had similar outcome scores. The proportions of students who reached criterion on the pretests for each problem type were extremely low and similar (see Table 3). Pretest proportions were not tested. On the posttest proportions, given in Table 3, Fisher's Exact Test was carried out. The significance levels, shown in Table 4, demonstrate that differences between instructed and comparison groups were reliable on all of representation, solution and answers for relational, proportional, and two-variable two-equation problems. In every case there were more students reaching criterion in the instructed group than the control group. The instructed students had reliably higher outcome scores on the instructed problems than the control students.

Because this was a pretest-posttest control group design with small numbers of subjects, outcome data for two other general problem solving measures were analyzed by means of analysis of covariance (Cook & Campbell, 1979). The covariate employed in each was the pretest score, in one case on a multiple-choice tests and, in the other case, an open-ended test. Preliminary analyses were carried out to ensure that the data met two assumptions required for the analysis of covariance: homogeneity of variance and parallel regression lines.

The analysis of covariance for adjusted posttest scores on the multiple-choice test did not meet the requirements of homogeneity of variance and parallel regression lines. A Krushal-Wallis non-parametric test was applied, yielding no significant differences. Inspection of the data suggests a ceiling effect (Cook & Campbell, 1979)) on this test, which had been designed for 8th Graders in British Columbia.

The analysis of covariance for adjusted posttest scores on the open-ended test met the required assumptions and showed that the difference between the means of the instructed and control groups was reliable $F(1,17) = 37.39$, $p < .05$. The instructed students had higher posttest scores in algebra word problem solving on the open-ended measure than the control students.

For two of the three measures used, the instructed students had significantly higher outcome scores than the control group. There were no significant differences on the third measure.

Significance of the Study

This study affirms the current models of problem solving which view problem solving as being composed of two phases: representation and solution (Mayer, 1985). It demonstrates that specific instruction in representation is feasible and effective, even for adolescents with learning disabilities. Both representation and solution appear to be essential for algebraic problem solving and both are amenable to intensive instruction that respects the complexity of domain-specific knowledge.

The isomorph (Simon & Hayes, 1976) or problem type used in this study is an effective and parsimonious vehicle for instruction of algebraic word problems. The isomorph is of problems having essentially the same mathematical relationships, having equations in the same form, having solution paths that map directly on to one another, and having cover stories that varied. The notion of isomorph has been refined to reflect the current emphasis on representation. The students' successes on the near- and far- transfer problems suggest that variations of surface and mathematical structures can be transferred once the schemata for the mathematical structure has been acquired at the level of isomorph.

The present study was conducted with learning disabled adolescents. This constitutes a stringent test of the instructability of representation. Furthermore, the study amply demonstrates the significance of applying cognitive theory to instruction for the learning disabled.

References

Brown, A. L., & Campione, J. (1984). Three faces of transfer: Implications for early competence, individual differences, and instruction. In M. Lamb, A. L. Brown, & B. Rogoff (Eds.), Advances in Developmental Psychology, Vol. 3 (pp. 143-192). Hillsdale, NJ: Erlbaum.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Lindquist, M. M., & Reyes, R. E. (1980). National assessment: A perspective of mathematics achievement in the United States. In R. Karplus (Ed.), Proceedings of the Fourth International Conference for the Psychology of Mathematics Education. Berkeley, CA: International Group for the Psychology of Mathematics Education.

Cook, T. D., & Campbell, D. T. (1979) Quasi-experimentation: Design and analysis issues for field settings. Boston: Houghton-Mifflin.

Ebos, F., Robinson, R., & Tuck, R. (1984). Math is/2. Toronto: Nelson.

Eicholz, R. E., O'Daffer, P. G., Brumfiel, C. F., Shanks, M. F., & Fleenor, C. R. (1971). School mathematics II. Toronto: Addison-Wesley.

Ericcson, K. A., & Simon, H. A. (1984). Protocol analysis: Verbal reports as data. Cambridge, MA: MIT Press.

Fleischner, J. E, & Garnett, K. (1983). Arithmetic difficulties among learning-disabled children: Background and current directions. Learning Disabilities, 2, 111-125.

Gagné, R. M., & Beard, J. G. (1978). Assessment of learning outcomes. In R. Glaser (Ed.), Advances in instructional psychology (Vol. 1) (pp.261-294). Hillsdale, NJ: Erlbaum.

Glaser, R. (1984). Education and thinking: The role of knowledge. American Psychologist, 39, 93-104.

Heller, J. I., & Reif, F. (1984). Prescribing effective human problem-solving processes: Problem description in physics. Cognition and Instruction, 1, 177-216.

Hinsley, D. A., Hayes, J. R., & Simon, H. A. (1977). From words to equations--meaning and representation in algebra word problems. In M. Just & P. Carpenter (Eds.), Cognitive processes in comprehension (pp. 89-106). Hillsdale, NJ: Erlbaum.

Hutchinson, N. L. (1985). A critical analysis and examination of research relevant to Glaser's "Education and thinking: The role of knowledge." Occasional Paper 85-02. Burnaby, BC: Simon Fraser University, Instructional Psychology Research Group.

Hutchinson, N. L. (1986). Instruction of representation and solution in algebraic problem solving with learning disabled adolescents. Unpublished doctoral dissertation, Simon Fraser University, Burnaby, B.C.

Learning Assessment Branch. (1985). Q2 British Columbia Achievement Test (Grade 10). Victoria, BC: Ministry of Education.

Lee, W. M., & Hudson, F. G. (1981). A comparison of verbal problem-solving in arithmetic of learning-disabled and non-learning disabled seventh grade males. (Res. Rep. No. 43). Lawrence, KS: University of Kansas, Institute for Research in Learning Disabilities.

Mayer, R. E. (1981). Frequency norms and structural analysis of algebraic story problems into families, categories, and templates. Instructional Science, 10, 135-175.

Mayer, R. E. (1985). Mathematical ability. In R. J. Sternberg (Ed.), Human abilities: An information-processing approach (pp. 127-150). New York: Freeman.

Meichenbaum, D. (1977). Cognitive behavior modification: An integrative approach. New York: Plenum.

Nuzum, M. (1983). The effects of an instructional model based on the information processing paradigm on the arithmetic problem solving performance of four learning disabled students. (Doctoral dissertation, Teachers College, Columbia University, 1983). Dissertation Abstracts International, 44, 1421-A.

Reif, F., & Heller, J. I. (1982). Knowledge structure and problem solving in physics. Educational Psychologist, 17, 102-127.

Robitaille, D. F., Sherrill, J. M., Kelleher, H. J., Klassen, J., & O'Shea, T. J. (1980). British Columbia Mathematics Achievement Tests, Grade 7/8 Applications. Victoria, BC: Ministry of Education.

Simon, H. A. (1980). Problem solving and education. In D. T. Tuma & F. Reif (Eds.), Problem solving and education: Issues in teaching and research (pp.81-96). Hillsdale, NJ: Erlbaum.

Simon, H. A., & Hayes, J. R. (1976). The understanding process: Problem isomorphs. Cognitive Psychology, 8, 165-190.

Sobel, M. A., & Maletsky, E. M. (1974). Mathematics I. Toronto, ON: Ginn.

Stein, E. I. (1982). Algebra in easy steps. Toronto, ON: Allyn & Bacon.

Table 1. Characteristics of students in the study

Group	S#	Age (Months)	Sex	Grade Placement	School	Score on Basic Operations	Score on 1-Operation Word Problems	Score on Instructional Word Problems
C	1	162	F	8	1	100	80	40
C	2	160	F	8	1	100	100	33
C	3	188	M	10	1	86	80	13
C	4	188	M	10	1	100	60	33
C	5	169	M	9	2	97	100	27
C	6	169	M	9	2	100	100	0
C	7	180	F	8	2	97	60	7
C	8	162	F	9	2	97	80	7
I	1	166	F	8	1	100	100	33
I	2	184	F	10	1	94	80	7
I	3	176	F	9	2	94	80	13
I	4	162	M	9	2	100	80	20
I	5	180	F	10	2	97	60	7
I	6	151	F	8	2	97	80	7
I	7	161	M	8	1	94	80	7
I	8	178	M	8	1	100	80	40
I	9	171	M	9	2	97	100	40
I	10	183	F	10	2	100	60	7
I	11	190	M	9	1	97	80	13
I	12	179	F	9	2	94	60	0

Note: The values for the three scores represent percentages correct.

Type of Problems	Representation	Solution	Answers
Relational	12/12	11/12	9/12
Proportion	10/10	10/10	10/10
Two-Variable Two-Equation	6/6	6/6	6/6

Table 2. Students reaching criterion on near-transfer problems

		Pretest			Posttest		
		R	S	A[a]	R	S	A
Relational Problems	Comparison Group	0/8	0/8	0/8	0/8	0/8	0/8
	Instructed Group	0/12	0/12	0/12	12/12	11/12	12/12
Proportion Problems	Comparison Group	0/8	0/8	3/8	0/8	1/8	3/8
	Instructed Group	0/12	0/12	2/12	10/12	11/12	11/12
Two-Variable Two-Equation Problems	Comparison Group	0/8	0/8	0/8	0/8	0/8	0/8
	Instructed Group	0/12	0/12	0/12	6/12	6/12	6/12

[a]R = Representation; S = Solution, A = Answer.

Table 3. Attainment of criterion at pretest and posttest

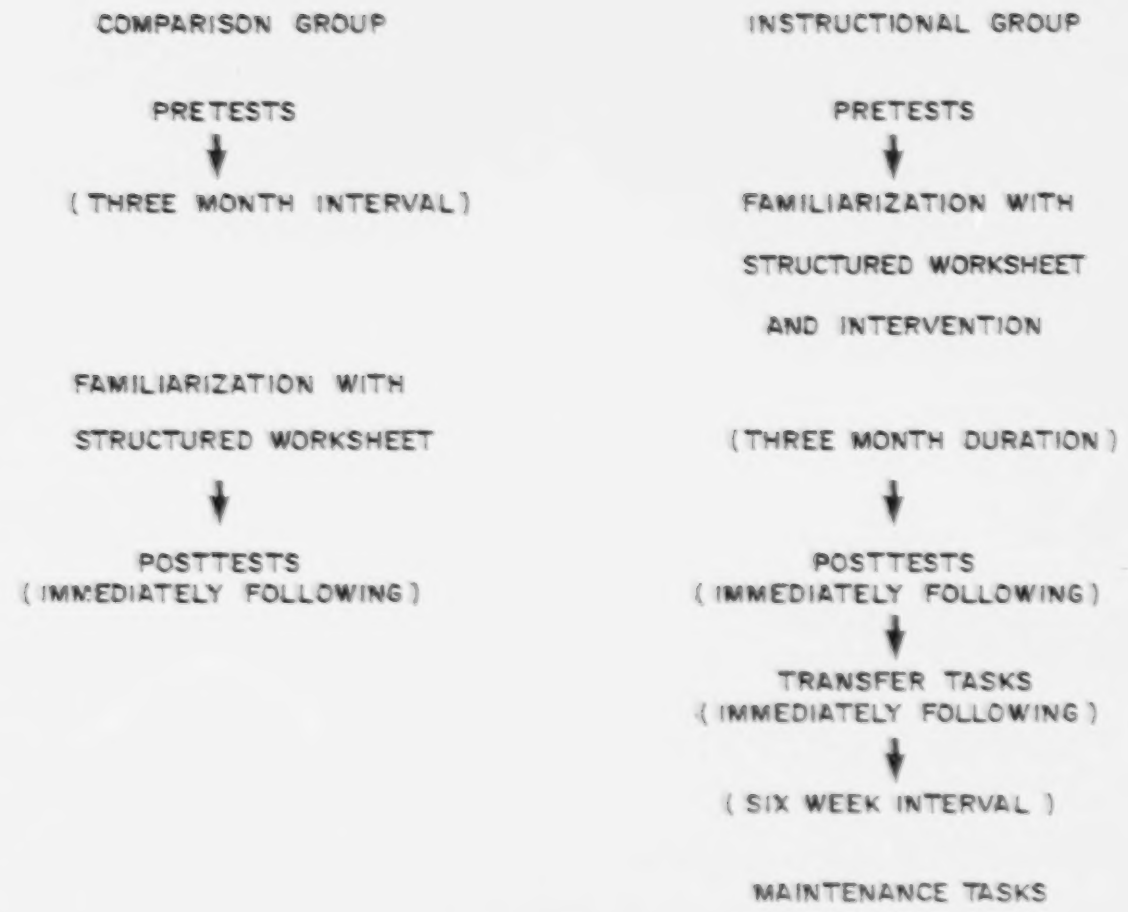

Figure 1. Two-group design

1. Mike has cycled 13 more km than Sam. Together they have cycled 121 km. Find the number of km Mike has cycled.

2. The sum of two numbers is eight and one of them is two greater than the other. Find the numbers.

3. Jean gave twice as much money to the Red Cross as did Ann. If the sum of their gifts was two dollars and thirteen cents, how much did each give?

4. Ellen is seven years older than her sister, and the sum of their ages is 21 years. How old is Ellen's sister?

5. A grocer packs 70 kilograms of cookies. There are 28 more kilograms of chocolate cookies than peanut butter cookies. Find the number of kilograms of chocolate cookies he packed.

Figure 2. Tasksheet #1. Relational problems

Problem _____ Date _____ Name _____

Exercise _____ Exp. Phase _____ Session _____ Ob. _____

Goal: _____

What I don't know: _____

What I know:

I can write/say this problem in my own words or draw a picture.

Kind of problem: _____

Equation:

_____

Solving the equation:

Solution:

[compare to goal]

Check:

Figure 3. Structured worksheet

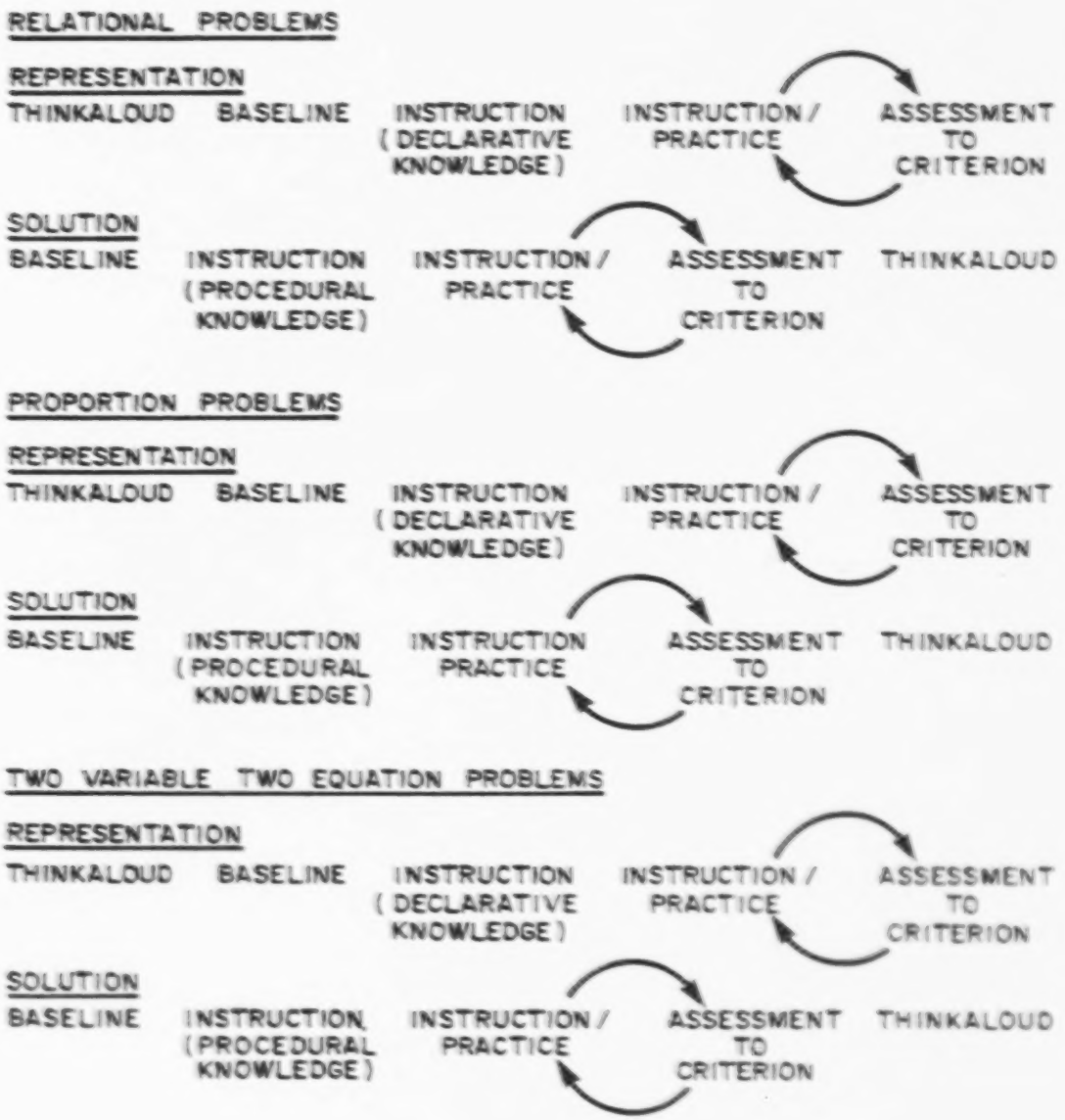

Figure 4. Successive phases in single-subject design

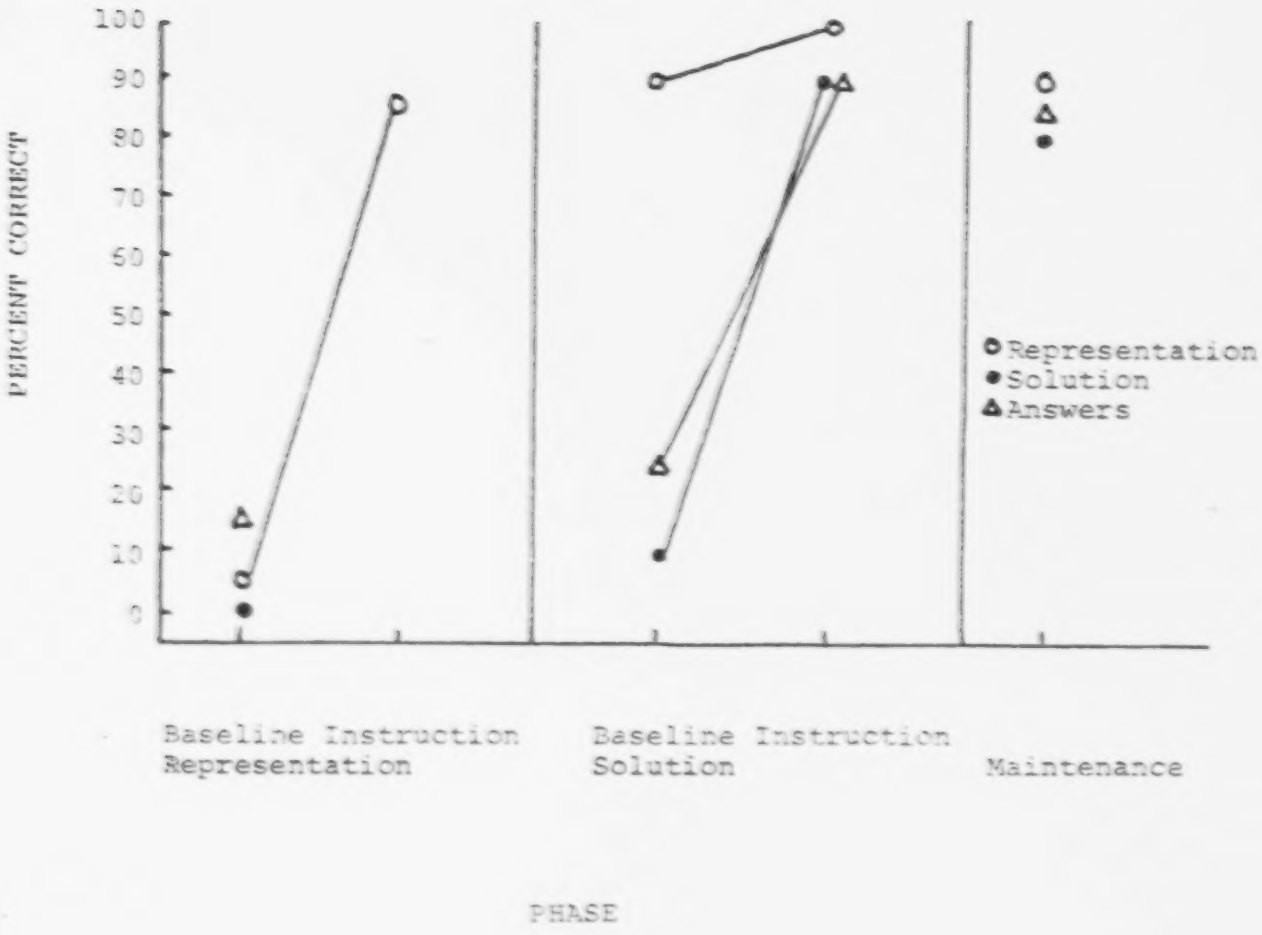

Figure 5. Relational problems correct in the phases of the study Representation and Solution

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